5 Inequalities and Descriptive Geometry

CHAPTER 5 Inequalities and Descriptive Geometry

In Chapter 1 your teacher asked you to think of a number and then do several mathematical operations with it to find a specific answer. In Section 5.1, you will be using mathematical operations to do mathematical "magic" tricks. You will also learn how the tricks work and be able to create your own tricks using variables and algebra tiles.

In Section 5.2, you will compare two expressions using algebra tiles on Expression Comparison Mats, discovering the legal moves that allow you to simplify expressions. Then you will determine which expression is greater (or if they are equal) and learn how to record solutions to inequalities using number lines with boundary points. Then your class will study the amazing number zero.

Guiding Questions

Think about these questions throughout this chapter:

How can I build it?

What's the relationship?

Are they equivalent?

Is there more than one way?

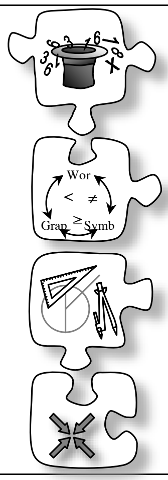
How can I measure it?

Section 5.3 has you explore different geometric shapes using a compass, a straightedge, and a protractor. You will do a circle walk and learn how to find the area of a circle.

In this chapter, you will learn how to:

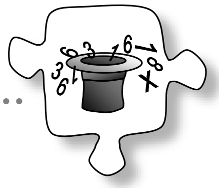
- Rewrite expressions by combining like terms and using the Distributive Property.
- Simplify and compare two algebraic expressions.
- Write and solve algebraic inequalities.
- Construct geometric shapes.
- Find the circumference and area of a circle.

Chapter Outline



Section 5.1	As you play math tricks, you will learn symbolic manipulation skills such as simplifying, combining like terms, distributing multiplication across addition, and making zeros.
Section 5.2	In order to compare expressions, you will build additional strategies to maintain equivalence and relationships between expressions. You will solve inequalities and represent their solutions on a number line.
Section 5.3	You will use a straightedge and a compass as tools to create geometric shapes. You will also investigate how to find the area and circumference of a circle.
Section 5.4	This section offers several problems to use what you have learned in the first five chapters of this course.

5.1.1 Why does it work?



Inverse Operations

Variables are useful tools to represent an unknown number. In some situations they represent a specific number, and in other situations they represent a collection of possible values. In previous chapters you have used variables to describe patterns in rules, to write lengths in perimeter expressions, and to define unknown quantities in word problems. In this chapter you will continue your work with variables and explore new ways to use them to identify specific values.

As you work in this chapter, you will often be called upon to **reverse** a process or operation in order to rewrite an expression or relationship. Applying this Way of Thinking by considering how to work in a different direction will help you understand several of the tools you will learn about in this chapter.

5-1. THE MATHEMATICAL MAGIC TRICK

Have you ever seen a magician perform a seemingly impossible feat and wondered how the trick works? Follow the steps below to participate in a math magic trick.

Think of a number and write it down.
Add five to it.
Double the result.
Subtract four.
Divide by two.
Take away your original number.

What did you get?

a. Check with others in your study team and compare answers. What was the result?

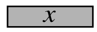
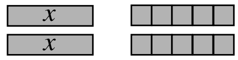
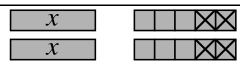
b. Does this trick work no matter what number you pick? Have each member of your team test it with a different number. Consider numbers that you think might lead to different answers, including zero, fractions, decimals and integers.

c. Which steps made the number you chose increase? When did the number decrease? What connections do you see between the steps in which the number got larger and the steps in which it got smaller?

d. How could this trick be represented with math symbols? To get started, think about different ways to represent just the first step, "Think of a Number."

5-2. Why does the magic trick from problem 5-1 work? Will the result always be the same?

To answer this question, Shakar decided to represent the steps with algebra tiles. Since he could start the trick with any number, he let an x-tile represent the "pick a number" step. With your team, analyze his work with the tiles. Then answer the questions below.

Steps	**Trial 1**	**Trial 2**	**Trial 3**	**Algebra Tile Picture**
1. Pick a number				
2. Add 5				
3. Double it				
4. Subtract 4				
5. Divide by 2				
6. Subtract the original number				

a. For the step "Add 5," what did Shakar do with the tiles?

b. What did Shakar do with his tiles to "double it?" Explain why that works.

c. How can you tell from his table that this trick will always end with 3? Explain why the original number does not matter.

5-3. The table below has the steps for a new "magic trick." Use the Lesson 5.1.1 Resource Page to complete parts (a) through (d) below.

Steps	**Trial 1**	**Trial 2**	**Trial 3**	**Algebra Tile Picture**
1. Pick a number				
2. Add 2				
3. Multiply by 3				
4. Take away 3				
5. Divide by 3				
6. Subtract the original number				

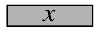
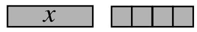
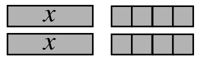
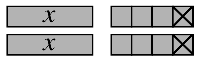
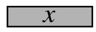
a. Pick a number and place it in the top row of the "Trial 1" column. Then follow each of the steps for that number. What was the end result?

b. Now repeat this process for two new numbers in the "Trial 2" and "Trial 3" columns. What do you notice about the end result?

c. Now use algebra tiles to see why your observation from part (b) works. Let an x-tile represent the number chosen in Step 1 (just like Shakar did in problem 5-2). Then follow the instructions with the tiles. Be sure to draw diagrams on your resource page to show how you built each step.

d. Explain how the algebra tiles help show that your conclusion in part (b) will always be true no matter what number you originally selected.

5-4. Now **reverse your thinking** to figure out a new "magic trick." Locate the table below on the Lesson 5.1.1 Resource Page and complete parts (a) through (c) below.

Steps	**Trial 1**	**Trial 2**	**Trial 3**	**Algebra Tile Picture**
Pick a number				
1.				
2.				
3.				
4.				
5.				

a. Use words to fill in the steps of the trick like those in the previous tables.

b. Use your own numbers in the trials, again considering fractions, decimals, and integers. What do you notice about the result?

c. Why did this result happen? Use the algebra tiles to help explain this result.

5-5. In the previous math "magic tricks," did you notice how *multiplication* by a number was later followed by *division* by the same number? These are known as **inverse operations** (operations which "undo" each other).

a. What is the inverse operation for addition?

b. What is the inverse operation for multiplication?

c. What is the inverse operation for "divide by 2?"

d. What is the inverse operation for "subtract –9?"

5-6. How does a trick (like the one from problem 5-1) work? You will answer this question by examining one more trick. In this last trick:

- Complete three trials using different numbers. Use at least one fraction or decimal.
- Use algebra tiles to help you analyze the trick, as you did in problem 5-3. Draw the tiles in the table on the resource page.
- Find at least two pairs of inverse operations in the process that are "undoing" each other.

Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number				
2. Double it				
3. Add 4				
4. Multiply by 2				
5. Divide by 4				
6. Subtract the original number				

Review & Preview

5-7. Write the inverse operations, that is, operations that "undo" one another for each situation below.

a. What is the inverse operation for "add $\frac{3}{4}$?"

b. What is the inverse operation for "subtract $1\frac{2}{3}$?"

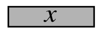
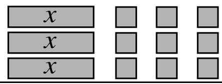
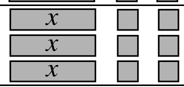
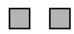
c. What is the inverse operation for "divide by 8?"

d. What is the inverse operation for "multiply by 12?"

5-8. Simplify the following expressions.

a. $1\frac{1}{2}+2\frac{1}{8}$ b. $\frac{4}{5}-\frac{2}{3}+\frac{1}{6}$ c. $5\frac{3}{5}-1\frac{4}{5}$

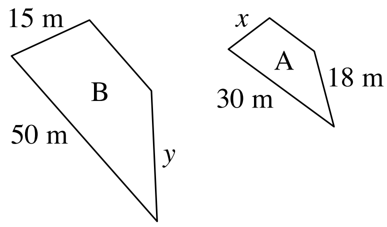
5-9. Draw the table below on your paper and look carefully at the algebra tiles in order to fill in each of the "Steps." Use your own numbers in the trials, again considering fractions, decimals, and integers.

Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number				
2. Add ____				
3. Multiply by ____				
4. Subtract ____				
5. Divide by ____				
6. Subtract the original number				

5-10. The trapezoids at right are similar shapes.

a. What is the scale factor between shapes A and shape B?

b. Find the lengths of the missing sides.



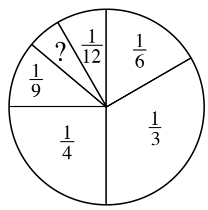
5-11. The Aloha Stadium in Honolulu, Hawaii, has seats for 50,000 people. At an upcoming football game, a company is planning to give away free hats to people based on where they are sitting.

a. The seats are divided into 40 different sections. If hats are given in only 5 sections, what is the probability of a guest sitting in a section that gets a hat?

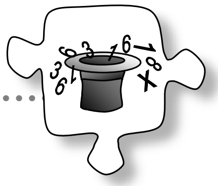
b. The company is going to choose three rows in each section to win the hats. There are 46 rows in a section. If you are sitting in a winning section, what is the probability that you are not sitting in a winning row?

c. The company plans to give away 750 hats. If you buy a ticket to the game, what is the probability you will receive a hat?

5-12. **Additional Challenge:** Robert found an old game in a closet and wanted to play it. However, a portion of the spinner shown at right could not be read. Find the missing portion of the spinner for Robert.



5.1.2 How can I represent it?



Translating Situations into Algebraic Expressions

In Lesson 5.1.1, you looked at how mathematical "magic tricks" work by using inverse operations. In this lesson, we will connect the algebra tile picture to another representation of the situation: the variable expression. Consider the following questions today:

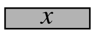
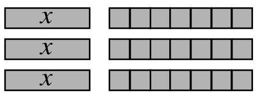
How can I **visualize** it?

How can I write it?

How can I express this situation efficiently?

5-13. Today you will consider a more complex math magic trick. Today the table you use to record your steps will have only two trials, but it will add a new column to represent the algebra tiles with a variable expression. Get a Lesson 5.1.2 Resource Page from your teacher.

- Work with your team to choose different numbers for the trials.
- Decide how to write variable expressions that represent what is happening in each step.

Steps	**Trial 1**	**Trial 2**	**Algebra Tile Picture**	**Variable Expression**
1. Pick a number				
2. Add 7				
3. Triple the result				
4. Add nine				
5. Divide by 3				
6. Subtract the original number				

5-14. For this number trick, the steps and trials are left for you to complete by using the variable expressions. To start, copy the table below on your paper and build each step with algebra tiles. Describe Steps 1, 2, and 3 in words.

Steps	Trial 1	Trial 2	Variable Expression
1.			x
2.			$x+4$
3.			$2(x+4)$
4.			$2x+20$
5.			$x+10$
6.			10

a. Look at the algebra tiles you used to build Step 3. Write a different expression to represent those tiles.

b. What tiles do you have to add to build Step 4? Complete steps 4, 5, and 6 in the chart.

c. Complete two trials and record them in the chart.

5-15. In Step 3 of the last magic trick (problem 5-14) you rewrote the expression $2(x+4)$ as $2x+8$. Can all expressions like $2(x+4)$ be rewritten without parentheses? For example, can $3(x+5)$ be rewritten without parentheses? Build it with tiles and write another expression to represent it. Does this work for all expressions?

5-16. Diana, Sam, and Elliot were working on two different mathematical magic tricks shown below. Compare the steps in their magic tricks. You may want to build the steps with algebra tiles.

Magic Trick A	**Magic Trick B**
1. Think of a number.	1. Think of a number.
2. Add three.	2. Multiply by two.
3. Multiply by two.	3. Add three.

a. Each student had completed one of the tricks. After the third step, Diana had written $2x+6$, Sam had written $2(x+3)$ and Elliot had written $2x+3$. Which expression(s) are valid for Magic Trick A? Which one(s) are valid for Magic Trick B? How do you know? Use tiles, sketches, numbers, and **reasons** to explain your thinking.

b. How are the two magic tricks different? How does this difference show in the expression used to represent each trick?

5-17. Parentheses allow us to consider the number of groups of tiles that are present. For example, when the group of tiles $x+3$ in problem 5-16 is doubled in Magic Trick A, the result can be written $2(x+3)$. However, sometimes it is more efficient to write the result as $2x+6$ instead of $2(x+3)$.

a. Show at least two ways to write the result of these steps:

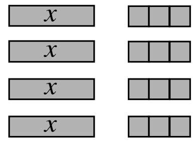
1. Think of a number.
2. Add five.
3. Multiply by three.

b. Write three steps that will result in $5(x-2)$.

c. How can the result from part (b) be written so that there are no parentheses?

5-18. At right is an algebra tile drawing showing the result of the first three steps of a number trick.

a. What could have been the three steps that led to this drawing?



b. Use a variable to write at least two expressions to represent the tiles in this problem, one of which contains parentheses.

c. If the next step in the trick is "Divide by 2," what should the simplified drawing and variable expression look like?

5-19. You have been writing expressions in different ways to mean the same thing. These expressions depend on whether you see tiles grouped by rows (like four sets of $x+3$ in problem 5-18) or whether you see separate groups (like $4x$ and 12). The **Distributive Property** is the formal name for linking these two equivalent expressions.

Write the following descriptions in two ways, one with parentheses and one without. For example, $4(x+3)$ can also be written $4x+12$.

a.
1. Think of a number.
2. Add 5.
3. Double it.

b.
1. Think of a number.
2. Add 7.
3. Multiply by 3.

5-20. **Additional Challenge:** Mrs. Baker demonstrated an interesting math magic trick to her class. She said,

"Think of a two digit number and write it down without showing me."

"Add the 'magic number' of 90 to your number."

"Take whatever digit is now in the hundreds place, cross it out and add it to the ones place. Now tell me the result."

As each student told Mrs. Baker their result, she quickly told them their original number. Why does this trick work? Consider the following questions as you unravel this trick:

a. Could you represent the original number with a variable? What is the algebraic expression after they add 90?

b. What is the largest number the students could have now? The smallest?

Problem continues on next page. →

5-20. *Problem continued from previous page.*

c. When the students cross off the 1 in the hundreds place (was any other number possible?), what have they subtracted from the expression? What is the new expression?

d. When the students add the 1 to the ones place, what is the simplified version of the expression they have created? What does Mrs. Baker mentally add to their result to reveal their original number?

METHODS AND MEANINGS

MATH NOTES

Algebraic Expressions

An **algebraic expression** consists of one or more variables, or a combination of numbers and variables possibly connected by mathematical operations. Each part of the expression separated by an addition or subtraction sign is called a **term**. A numerical term in an algebraic expression (like the 7 and 9 below) is called a **constant**. Expressions do not contain an equal sign (=). Four examples of algebraic expressions are:

$$4x\ ,\ 4x-3y+7\ ,\ \frac{3x^2}{2x+5}-9\ ,\ 3x-5(x^2+2)+1$$

In the examples above, the expressions have one term, three terms, two terms, and three terms, respectively. Algebraic expressions may be simplified or, if the values of the variables are known, expressions may be evaluated. See the examples below.

Simplify this expression:

$4x+5-2x+1$
becomes
$2x+6$

Evaluate the expression $3y+y^2$ for $y=-2$.

$3(-2)+(-2)^2$
$-6+4$
-2

Review & Preview

5-21. Copy the chart on your paper, then complete two trials by reading the variable expressions. Write in the steps as well.

Steps	Trial 1	Trial 2	Variable Expression
1.			x
2.			$6x$
3.			$6x+24$
4.			$6x+18$
5.			$x+3$
6.			3

5-22. Translate each of these situations into a variable expression such as those found in a magic number chart.

a. Think of a number and multiply it by seven.

b. Think of a number and divide it by eight.

c. Think of a number and reduce it by ten.

d. Think of a number, add two, then multiply by five.

5-23. Ms. Poppy has finished grading her students' tests. The scores were: 62, 65, 93, 51, 55, 76, 79, 85, 55, 72, 78, 83, 91, and 82.

a. Find the median.

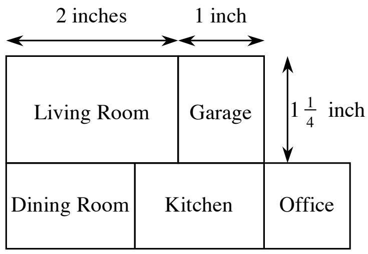
b. Find the range.

c. Find the mode.

d. Find the quartiles.

e. Find the mean.

5-24. The scale drawing at right shows the first floor of a house. The actual dimensions of the garage are 20 feet by 25 feet. All angles are right angles.



a. How many feet does each inch represent?

b. What is the length and width of the living room in inches?

c. What is the length and width of the living room in feet?

d. What is the area of the living room (in square feet)?

e. What is the perimeter of the garage (in feet)?

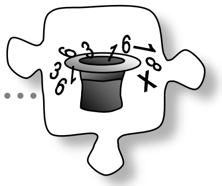
5-25. Graph the following points on a coordinate grid: $A(1,1)$, $B(2,3)$, $C(5,3)$, $D(5,1)$. Connect the points, including the last one to the first point.

a. What shape have you created?

b. What point can you move to create a rectangle? What would be the new coordinate?

c. Reflect trapezoid *ABCD* across the *y*-axis. What are the new coordinate points of the trapezoids vertices?

5.1.3 How can I simplify it?



Simplifying Algebraic Expressions

In the previous lesson, you represented more complex mathematical tricks with variable expressions instead of algebra tiles because the expressions were more efficient. In this lesson, you will explore various ways to make expressions simpler by making parts of them zero.

Zero is a relative newcomer to the number system. Its first appearance was as a placeholder around 400 B.C. in Babylon. The Ancient Greeks philosophized about whether zero was even a number: "How can nothing be something?" East Indian mathematicians are generally recognized as the first culture to represent the quantity zero as a numeral and number in its own right about 600 A.D.

Zero now holds an important place in mathematics both as a numeral representing the absence of quantity as well as a placeholder. Did you know there is no year 0 in the Gregorian calendar system (our current calendar system of 365 days in a year)? Until the creation of zero, number systems began at one. Consider the following questions as you work today:

How can I create a zero?

How can I rewrite this expression in the most efficient way?

5-26. CONCEPTS OF ZERO

Zero is a special and unusual number. As you read above, it has an interesting history. What do you know about zero mathematically? The questions below will test your knowledge of zero.

a. If two quantities are added and the sum is zero, what do we know about the quantities?

b. If we add zero to a number, how does the number change?

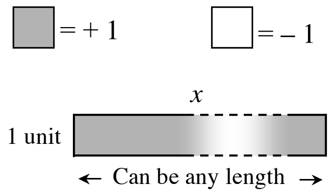
c. If we multiply a number by zero, what do we know about the product?

d. What is the opposite of zero?

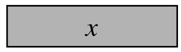
e. If three numbers have a product of zero, what do we know about at least one of the numbers?

f. Is zero even or odd?

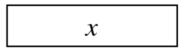
5-27. As you know, +1 is represented with algebra tiles as a shaded small square and is always a positive unit. The opposite of 1, written –1, is an open small square and is always negative. Is this true with the variable x-tiles too?



a. The variable x-tile is shaded, but is the number represented by a variable such as x always positive? Why or why not?



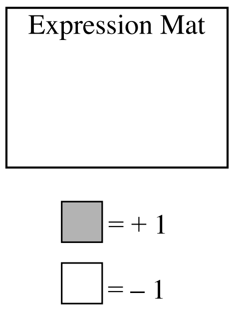
b. The opposite of the variable x, written $-x$, looks like it might be negative, but since the value of a variable can be any number (the opposite of –2 is 2), what can you say about the opposite of the variable x?



c. Is it possible to determine which is greater: x or $-x$? Explain.

d. What is true about $6+(-6)$? What is true about $x+(-x)$ (the sum of a variable and its opposite)?

5-28. Get a Lesson 5.1.3 Resource Page, from your teacher, which is called "Expression Mat." This will help you so you can tell the difference between the expression you are working on and everything else on your desk.



From your work in problem 5-27, we can say that situations like $6+(-6)$ and $x+(-x)$ create "zeros," that is, when we add an equal number of tiles and their opposites, the result is zero. The pairs of unit tiles and x tiles shown in that problem are examples of "zero pairs" of tiles.

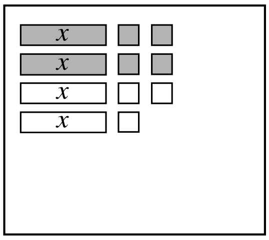
Build each collection of tiles represented below on the mat. Name the collection using a simpler algebraic expression (one that has fewer terms) by finding and removing zero pairs and combining like terms.

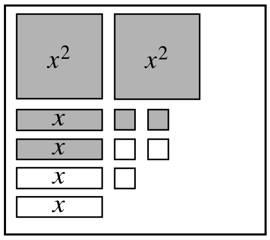
a. $2+2x+x+(-3)+(-3x)$

b. $-2+2x+1-x+(-5)+2x$

5-29. An **equivalent expression** refers to the same amount with a different name.

Build the expression mats shown in the pictures below. Write the expression shown on the expression mat, then write its simplified equivalent expression by making zeros (zero pairs) and combining like terms.

a. 

b. 

5-30. Build what is described in words below on your expression mat. Then write two different equivalent expressions to describe what is represented. One of the two representations should include parentheses.

a. The area of a rectangle with a width of 3 units and a length of $x+5$.

b. Two equal groups of $3x-2$.

c. Four rows of $2x+1$.

d. A number increased by one, then tripled.

5-31. Copy and rewrite the following expressions by combining like terms and making zeros. Using algebra tiles may be helpful.

a. $(-1)+(-4x)+2+2x+x$

b. $2x+4+(-3)+3x$

c. $3x^2+(-2x^2)+5x+(-4x)$

d. $x-6+4x+5$

5-32. **Additional Challenge:** Division by zero is an interesting concept. Some students believe that when you divide by zero, the result is zero. Consider how the numerical equation $30 \div 6 = 5$ can be **reversed** to make the multiplication equation $5 \cdot 6 = 30$.

a. **Reverse** the division equation $24 \div 4 = 6$ into a multiplication equation.

b. Now take the equation $24 \div 0 = x$ and **reverse** it into a multiplication equation. What value of x can make this equation true? Explain.

c. Why do you think the solution to a number divided by zero is called "undefined?"

d. Is $0 \div 24$ also "undefined?" Why or why not?

5-33. LEARNING LOG

In your Learning Log, explain how to simplify expressions in your own words. Include an example of different ways to write the expression $3(x+4)$ by drawing pictures and writing equivalent expressions. Title this entry "Simplifying Expressions" and include today's date.

MATH NOTES

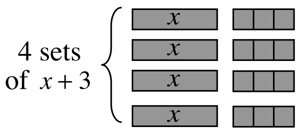
METHODS AND MEANINGS

Distributive Property

The **Distributive Property** states that multiplication can be "distributed" as a multiplier of each term in a sum or difference. It is a method to separate or group quantities in multiplication problems. For example, $3(2+4)=3\cdot 2+3\cdot 4$. Symbolically it is written:

$$a(b+c)=ab+ac \quad \text{and} \quad a(b-c)=ab-ac$$

For example, the collection of tiles at right can be represented as 4 sets of $x+3$, written as $4(x+3)$. It can also be represented by 4 x-tiles and 12 unit tiles, written as $4x+12$.



$4(x+3)=4x+12$

Review & Preview

5-34. Sketch each collection of tiles below. Name the collection using a simpler algebraic expression, if possible. If it is not possible to simplify the expression, explain why not.

a. $(-x)+7+4x+x+(-3)+(-3x)$

b. Seven plus three times a number, plus three minus three times the number.

c. Two groups of a number plus three.

d. $7+4x^2+5x$

e. $x^2+x^2+(-5)+2$

5-35. What number do you get with this number magic trick? Support your answer with at least three trials.

1. Think of a number
2. Add the next higher number
3. Add nine
4. Divide by two
5. Subtract your original number

5-36. Alden found a partially completed 5-D chart:

	Define			Do	Decide Target 74
Trial 1:	15	$2(15)=30$	$15+2=17$	$15+30+17=$	62 too small
Trial 2:	18	$2(18)=36$	$18+2=20$	$18+36+20=$	74 just right

a. Create a word problem that could have been solved using this chart.

b. What words would you put above the numbers in the three empty sections in the "Trial" and "Define" parts of the chart?

c. What word(s) would you put above the "Do" column?

5-37. Rachel is collecting donations for the local animal shelter. So far she has collected \$245, which is 70% of what she hopes to collect. How much money does Rachel plan to collect for the shelter? Show your work.

5-38. Simplify each expression.

a. $1.2-0.8$

b. $-4-(-2)$

c. $-\frac{6}{11}-(-\frac{1}{4})$

d. $\frac{2}{3}\cdot\frac{2}{5}$

e. $0.6\cdot 8$

f. $-\frac{5}{4}\cdot\frac{8}{13}$

5-39. Alex is trying to simplify the expression $-1\frac{1}{4}+(2\frac{1}{2})+(3\frac{1}{4})$. He started by rewriting it like this:

$$-1+(-\tfrac{1}{4})+2+\tfrac{1}{2}+3+\tfrac{1}{4}$$
$$(-1+2+3)+(-\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{2})$$

a. Why might he have regrouped the expression in this way?

b. Simplify the expression. What is the result?

c. What property was Alex using when he rewrote the problem?

d. Use this strategy to regroup the expression $\frac{3}{10}+2\frac{1}{10}+(-1\frac{2}{5})$ and find the result.